Teaching Multiple Concepts to a Forgetful Learner

**Motivating Applications**
- Language learning apps used by over 300 million students
- Based on spaced repetition technique
- Spacing effect: practice should spread out over time
- Lag effect: spacing between practices should gradually increase
- No known guarantees on scheduling multiple concepts over fixed horizon
- Key research problem that we tackle in this paper is:
  
  Can we compute near-optimal personalized schedule of repetition?

**Teaching Interaction using Flashcards**

Interaction at time $t = 1, 2, \ldots T$

1. Learner displays a flashcard $x_i \in \{1, 2, \ldots, n\}$
2. Learner’s recall is $y_i \in \{0, 1\}$
3. Teacher provides the correct answer

**Background on Teaching Policies**

Example setup

- $T = 20$ and $n = 5$ concepts given by $(a, b, c, d, e)$

Naïve teaching policies

- Random: $a = b = c = d = e$
- Round-robin: $a = b = c = d = e$

Key limitation: Schedule agnostic to learning process

Pimsleur method (1967)

- Used in mainstream language learning platforms
- Based on spaced repetition ideas

Key limitation: Non-adaptive schedule ignores learner’s responses

Leitner system (1972)

- Adaptive spacing intervals

**Learner: Memory Model and Responses**

- Half-life regression (HLR) model [Settles, Meeder'16]
  
  - Denote history up to time $t$ as $(x_{i1}, x_{i2}, \ldots, x_{it})$
  
  - Last time step when concept $x$ was taught is $t_f^x \in \{1, 2, \ldots, t\}$
  
  - Learner’s mastery for concept $x$ at time $t$ is $h_t^x$
  
  - Recall probability based on exponential forgetting: $g^x(t, (x_{i1}, x_{i2}, \ldots)) = 2^{-\Delta t / \lambda^x}$
  
  - Changes in half-life $h^x$ parameterized by $(a^x, b^x)$

**Teacher: Scheduling as Optimization**

Teacher’s objective function

- Given a sequence of concepts and observations $x_{1T}, x_{2T}, \ldots$, we define
  
  $$f(x_{1T}, y_{1T}) = \frac{1}{M} \sum_{t=1}^{T} g^x(t + 1, (x_{i1}, x_{i2}, \ldots))$$

  Area under the curve

Optimization problem

- Teaching policy is given by $\pi: (x_{i1}, x_{i2}, \ldots) \rightarrow \{1, 2, \ldots, n\}$
- Average utility of a policy $\pi$ is $F(\pi) = \mathbb{E}_{i \sim \pi} [f(x_{IT}, X_{IT})]$
- Optimal policy is given by $\pi^* = \arg \max_{\pi} F(\pi)$

Adaptive greedy algorithm

- for $t = 1, 2, \ldots, T$
  
  - Select $x_i = \arg \max_{x \in \{1, 2, \ldots, n\}} \mathbb{E}_{i \sim \pi} [f(x_{i1}, x_{i2}, \ldots, y_i)] - f(x_{i1}, x_{i2}, \ldots)$
  
  - Observe learner’s recall $y_i \in \{0, 1\}$
  
  - Update $x_{i+1} \leftarrow x_{i1} \oplus x_{i2} \oplus \ldots \oplus x_i \oplus y_i$

Characteristics of the problem

- Non-submodular
  
  - Gain of a concept $x$ can increase given longer history
  
  - Captured by submodularity ratio $\gamma$ over sequences

- Post-fix non-monotone
  
  - $f_{\text{orange}}(x) \leq f_{\text{blue}}(x)$
  
  - Captured by curvature $\omega$

**Theoretical Guarantees**

Guarantees for general case (any memory model)

- Utility of $n^k$ (greedy policy) compared to $n^{opt}$ is given by
  
  $$F(n^k) \leq F(n^{opt}) \leq F(n^k) + \frac{1}{\omega^{\max}} (1 - e^{-\omega^{\max}/\gamma})$$

Guarantees for the HLR model

- Theorem 5. Consider the task of teaching $n$ concepts where each concept is following an independent HLR model with the same parameters $(a^x, b^x)$ for $x \in \{1, 2, \ldots, n\}$.
- A sufficient condition for the algorithm to achieve $(1-\epsilon)$ high utility is $x \geq \max \{\log T, \log(2n), \log \left(\frac{C}{\omega^{\max}}\right)\}$.

Illustration

- $T = 15$ and $n = 3$ concepts using HLR model with different parameters

**Results on Human Participants**

Online learning platforms

- German vocabulary for language learning: https://www.teaching-german.co/
- Recognizing animal species from images: https://www.teaching-biodiversity.co/

Experimental setup

- Performance measured by gain in knowledge: postquiz score – prequiz score
- $T = 40$, $n = 15$; participants from a crowdsourcing platform (80 and 320)
- Dataset of 100 English-German word pairs
- Dataset of 50 animal images of common and rare species

Algorithms

- GR: Our algorithm; RD: Random; RR: Round-robin
- LR: Least-recall (generalization of Pimsleur method and Leitner system)

- **German**
  - Avg. gain $0.572$; $0.487$; $0.462$; $0.467$
  - $p$-value $0.0652$; $0.1970$; $0.0151$

- **Biodiversity (all species)**
  - Avg. gain $0.475$; $0.411$; $0.390$; $0.251$
  - $p$-value $0.0017$; $0.0001$; $0.0001$

- **Biodiversity (rare species)**
  - Avg. gain $0.766$; $0.668$; $0.601$; $0.376$
  - $p$-value $0.0001$; $0.0001$; $0.0001$